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PG I Semester

Unit - 4

Topic :- Properties of Group

Properties of Groups

Let $(G, *)$ be a group, then

(1) The identity element 'e' is unique.

(2) There exists unique inverse in G i.e. $a^{-1} \in G \ \forall \ a \in G$.

Also $(a^{-1})^{-1} = a \ \forall \ a \in G$ and $(a * b)^{-1} = b^{-1} * a^{-1} \ \forall \ a, b \in G$.

In general

If $a_1, a_2, \ldots a_n$, are elements of a group G, then $(a_1 * a_2 * \cdots * a_n)^{-1}$

$$= a_n^{-1} * a_{n-1}^{-1} * \cdots * a_2^{-1} * a_1^{-1}.$$

(3) (i) $a * c = b * c \Rightarrow a = b$ (Right cancellation law)

(ii) $c * a = c * b \Rightarrow a = b$ (left cancellation law)

where $a, b, c \in G$

(4) The left identity is also the right identity.

i.e. $ea = a$ and $ae = a$

(5) The left inverse of an element is also its right inverse.

i.e. $a^{-1}a = e$ and $aa^{-1} = e$

(6) The equ^{n}s $a * x = b$ and $y * a = b$, where $a, b \in G$ have unique sol^{n}s in G, which are $x = a^{-1} * b \in G$ and $y = b * a^{-1} \in G$ respectively.

<u>Note</u> With the help of the above theorem, we can define the group alternatively as follows.

A set G with a binary composition $*$ is a group iff.

(i) The composition $*$ is associative.

(ii) The equ^{n}s $ax = b$ and $ya = b$ have unique sol^{n}s in G.

or (i) Left identity and left inverse exists in group.

or (ii) Right identity and right inverse exist in group.

or A finite set G, with a binary operation $*$ which is associative is a group iff the cancellation laws held.

9 ⊛ The result of the above theorem may not hold when G is an
10 infinite group.
e.g., In the set N of natural numbers,
11 for the binary operation +, associative and cancellation laws also held. But
12 (N, +) is not a group.